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Computation of Electric Field from Lighting Discharges

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Abstract: The electric fields due to lightning at ground was evaluated for static cases using the coulombs law. The concept of electrical image was developed considering ground as a conducting grounded plane. A tri-pole model for the charge structure of thundercloud was developed and electric fields due to such cloud structure were evaluated. Variations of electric fields due to distance were assessed and the expression of reversal distance was introduced. The graph describing the reversal distance from lightning was introduced.

1. Introduction

A sudden electrostatic discharge that occurs between electrically charged regions between cloud-to-cloud, cloud- to -air, and cloud- to -ground is generally referred as lightning. (Nag & Rakov, 2010), described that intra cloud discharges are cloud lightning discharges that produce single bipolar electric field pulses sometimes referred as Narrow Bipolar Pulses. (Uman M. A., 2001), described that lightning is a transient, high current electric discharge whose path length is measured in km and the most common sources of lightning is the electric charge separated in ordinary thunderstorm clouds(cumulonimbus). In order for an electric discharge to occur two conditions are of maximum importance, they are a sufficiently high electric potential between two regions of space and a high resistance medium must obstruct the equalization of the opposite charges. As the thundercloud moves over the surface of the earth which is considered as an infinitely conducting grounded plane an equal electric charge of opposite polarity is induced on the surface of the earth. The oppositely charged create an electric field within the air between them. The greater the accumulation of charges, higher will be the electric field. The electric field created due to lightning is a complex process to comprehend. (Williams, 1989), described that the origin of electrification in cloud is still not properly understood and earlier reviews of this subject are often biased in favor of particular mechanism for electrification. In general, the overall length of the lightning discharge is considered as a very thin channel. The calculation of electric and magnetic fields from different lightning process has more practical application. The electromagnetic fields from the lightning phenomena can couple the electrical system and produce transient over voltage which destroy the electronics. In this case the lightning can cause power and telecommunication outages and destroy the electronic devices. So, it is more important and more applicable. The electric and magnetic fields are derived for simple charge and current configuration at the static case (without varying fields) applicable to the lightning phenomena. The goal of this paper is to compute the electric field due to lightning and analyze the variations of electric fields at ground with distance.

2. Electrostatic Field from a Dipole

From coulombs law, the electric field E at a point P at a distance r, from a point charge Q in a medium of dielectric constant K is $E = \frac{Q}{Kr^2}$, In most of the cases of atmospheric electricity the medium is air whose dielectric constant is approximately unity. So, the electric field intensity is given by $\vec{E} = \frac{Q}{r^3}.\vec{r}$ …… (2)

The direction of the electric field as well as force pointing away from Q if it is positive charge and pointing towards it if Q is a negative charge as shown in the above figure. The electric potential can be defined as the amount of work done in bringing unit positive charges from infinity (out of field) to that point. From this, it is obvious that we cannot calculate the absolute potential at a given point. For convenience, we consider the surface of the earth to be at zero potential. The electric field intensity E may be defined as the negative rate of change of potential with respect to distance. Mathematically, $\vec{E} = -\frac{dV}{dr}$ …… (3) along the direction of r.

The negative sign of the equation implies that the potential increases with distance in a direction opposite to the direction of the field E. The above equations thus not only signify that the respective vectors E and gradient of V are equal in absolute magnitude but also that they have opposite direction which is shown in the figure 1.

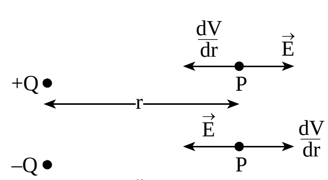

Fig.1: the electric field E and the potential gradient $\frac{dV}{dr}$ at a point

The electric field close to the surface (of a conductor in which no current is flowing) is perpendicular to the surface. Since the earth is a conductor, the electric field above the level ground is vertical. The earth is negatively charged, and the conducting layers of the upper atmosphere have a net positive charge. It follows therefore that during fine weather, the electric field remains vertical up to high altitudes then the potential gradient is a vector pointing upwards. For the sign convention, it is useful to remember that a positive charge in a cloud will contribute a positive component to the field at a ground level and a negative charge in a cloud will contribute a negative component to the field at ground level. The –Q charge is called the electrical image of +Q. If +Q represents a charge in a thundercloud at a height H above the earth's surface the same simple procedure of calculation is valid, since the surface of the earth may be considered to be a conducting plane, its curvature being negligibly small over the context of the distances usually involved.

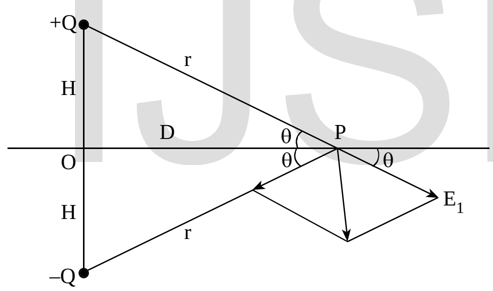

Fig.2: electric field E produced by the point charged +Q at point P
where OP is the surface of the ground

Let us consider a +Q charge on the cloud at height H above from the surface of the ground OP, then –Q be the electrical image charge, whose distance is also H from OP line. Let us consider a point P at a distance D from O, at which the electric field should be determined. The electric field at the point P due to the Q charge is $\vec{E}_1 = \frac{Q}{r^2}\hat{r}$. Similarly, the electric field at the same point P due to the –Q charge (electrical image charge) is $\vec{E}_2 = \frac{Q}{r^2}\hat{r}$ Since the direction of E_1 and E_2 as shown in figure, then the total electric field is $\vec{E} = \vec{E}_1 + \vec{E}_2$. Let θ be the angle made by the r with the surface of the ground as shown in figure. Then, the total electric field is $E = \frac{2Q\sin\theta}{r^2}$ ……………..…… (4)

(As the components of E_1 and E_2 in the direction of E are both given by $\frac{Q\sin\theta}{r^2}$ and $E_1 = E_2$. Similarly, another components $E_1 \cos\theta$ in opposite direction and they cancelled each other.

The direction of this electric field is perpendicular to the surface of the ground.

From the figure, $r^2 = H^2 + D^2$; $\sin\theta = \frac{H}{r}$ $\cos\theta = \frac{D}{r}$ ………..(5)

From equation (4) and (5) then, we get, $E = \frac{2QH}{(H^2 + D^2)^{3/2}}$ ……………….. (6)

Similarly, if –Q charge is taken in the cloud at the height H above from the surface of the ground, the process is similar and we get the same result .But by the convention, the electric field at point P is positive field if the charge in the cloud is positive and the electric field at point P is negative field if the charge in the cloud is negative.

In SI system the equation 6 can be written as, $E = \frac{2QH}{4\pi\epsilon_0 (H^2 + D^2)^{3/2}}$ …… (7)

Substituting $\frac{H}{D} = x$ then, we get, $E = \frac{2Q.x}{4\pi\epsilon_0 D^2 (1+x^2)^{3/2}}$ …… (8)

This equation (8) gives the total electric field in magnitude with a direction perpendicular to the plane and positive field if the charge above the plane is positive and the negative field if the charge above the plane is negative. The total electric field is maximum when $\frac{dE}{dx}$ should be zero.

$$\text{So, } \frac{dE}{dx} = \frac{2Q}{4\pi\epsilon_0 D^2}\frac{d}{dx}[x(1+x^2)^{-3/2}] = \frac{2Q}{4\pi\epsilon_0 D^2}\frac{(1-2x^2)}{(1+x^2)^{5/2}}$$

For maximum value, this equation should be zero so, $1 - 2x^2 = 0$ that means, $\frac{H}{D} = \frac{1}{\sqrt{2}}$

By drawing the graph of electric field intensity with the value of x (i.e.$x=\frac{H}{D}$) shown in fig 3.

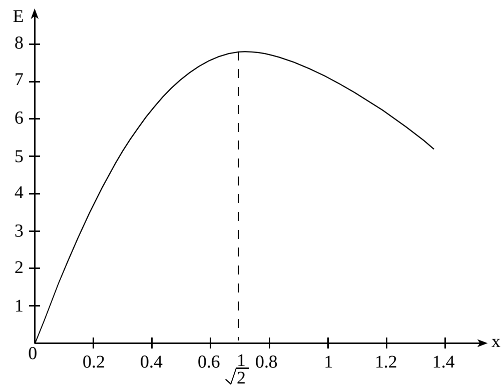

Fig.3: the graph of electric field intensity and x

If the height H of the cloud is varied, the electric field at the point P at a distance D will pass through a maximum as shown in the graph above at which $x = \frac{H}{D} = \frac{1}{\sqrt{2}}$.Then, following cases are possible. For $x << 1$ (is very small), then x^2 is also very small. Hence $\left(\frac{H}{D}\right)^2$ can be neglected. So, the total electric field (E) = $\frac{2QH}{4\pi \in_0 D^3} = \frac{M}{4\pi \in_0 D^3}$ …… (9)

Where M =2QH is the electric dipole moment of the charge Q and its image.

3. Simple Models for the Charges Transfer of the Overall Flash

A model for the charge structure of a thundercloud was developed in the early periods of 1930s. The model was derived from ground based measurement of both the electric field associated with the static cloud charges and the electric field changes associated with the effective neutralization of portion of those cloud charges by lighting as discussed in the historical review by (Uman & McLain, 1969). (Williams, 1989) in this paper describes that the observed tri-pole structure is quantitatively consistent with laboratory simulations and suggests that the collision between the ice crystals and rimed graupel particles is the dominant mechanism for the charge separation in thunder storms. Let us consider Q_P be the positive charged centre at a height of 10 km the ground and Q_N be the negative charged centre at a height of 5 km and q_p at a height of 2 km. The

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configuration of the primary charge structure in a thundercloud is that of a dipole: a lower negative charge Q_N and an upper positive charge Q_P as shown in figure 4.

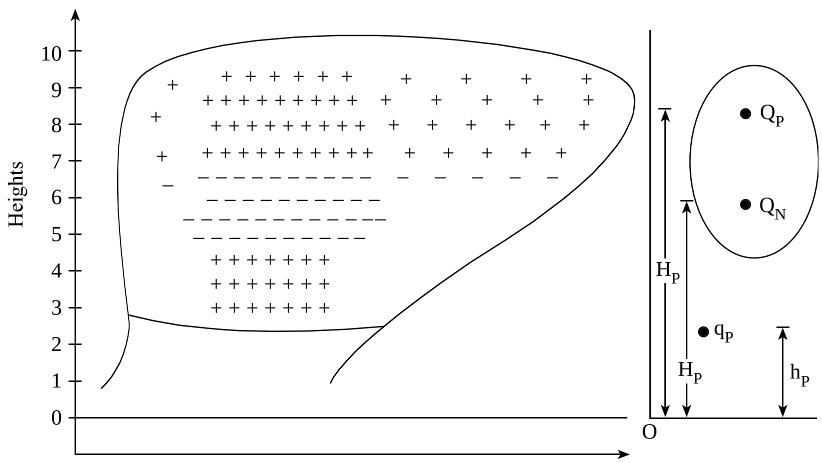

Fig.4: tri-pole structure of cloud

Let us consider the main positive charge $+Q_P$ centered at a point at a height of H_P, the main negative charge $-Q_N$ centered at a point at a height of H_N, the horizontal distance of main positive charge centre from the measuring station is D_P and the horizontal distance of main negative charge center from the measuring station is D_N. Then the electric field at the measuring station on ground given by positive charge is

$$E_+ = \frac{1}{4\pi \in_0} \cdot \frac{2Q_P H_P}{r^3} = \frac{1}{4\pi \in_0} \cdot \frac{2Q_P H_P}{(D_P{}^2 + H_P{}^2)^{3/2}} \quad \ldots\ldots (10)$$

directed perpendicularly towards the earth (i.e. inside the earth)

Similarly, the electric field at the same point given by the negative charge is

$$E_- = \frac{1}{4\pi \in_0} \cdot \frac{2Q_N H_N}{(D_N{}^2 + H_N{}^2)^{3/2}} \quad \ldots\ldots (11)$$

directed perpendicularly upwards from the earth.

Hence, the total electric field at the ground is $E = E_+ + E_-$

$$= \frac{1}{4\pi \in_0} \left[\frac{2Q_P H_P}{(D_P{}^2 + H_P{}^2)^{3/2}} - \frac{2Q_N H_N}{(D_N{}^2 + H_N{}^2)^{3/2}} \right] \quad \ldots\ldots (12)$$

From the general features of the electrostatic field from such a cloud charge distribution, we can consider the dipole charges are equal i.e. $Q_P = Q_N = Q$ and these dipoles are in vertical line that means the horizontal distances of positive charge centre and negative charge centre are at equidistant from the measuring station i.e. $D_P = D_N = D$. Thus, equation 12 can be written as

$$E = \frac{1}{4\pi \in_0}\left[\frac{2QH_P}{(D^2 + H_P{}^2)^{3/2}} - \frac{2QH_N}{(D^2 + H_N{}^2)^{3/2}}\right] \qquad \ldots\ldots (13)$$

The electric field of equation (13) is plotted as a function of D is represented as,

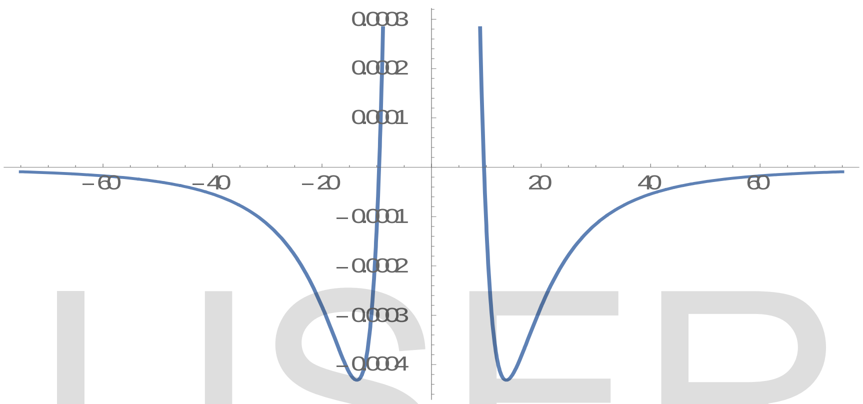

Fig.5: plot as a function of D

The most interesting feature of the field is that it reverses sign with range; it is negative at close distance and positive at faraway as shown in the graph. The distance at which the sign of the field changes i.e. the direction reverse is called reversal distance. Let D_0 be the reversal distance at which the field passes through zero. So from equation (13)

$$\frac{H_P}{(D^2 + H_P{}^2)^{3/2}} = \frac{H_N}{(D^2 + H_N{}^2)^{3/2}},$$

On taking $\frac{2}{3}$ in each power

$$H_P{}^{2/3}(D^2 + H_N{}^2) = H_N{}^{2/3}(D^2 + H_P{}^2)$$

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$$\text{Or, } D^2 = \frac{(H_N H_P)^{2/3}(H_P^{4/3} - H_N^{4/3})}{H_P^{2/3} - H_N^{2/3}}$$

$$= \frac{(H_N H_P)^{2/3}(H_P^{2/3} - H_N^{2/3})(H_P^{2/3} + H_N^{2/3})}{H_P^{2/3} - H_N^{2/3}}$$

$$\text{Or,} D^2 = (H_N H_P)^{2/3} (H_P^{2/3} + H_N^{2/3})$$

Hence the reversal distance (D_0) = $(H_N H_P)^{1/3} (H_P^{2/3} + H_N^{2/3}]^{1/2}$ …… (14)

4. Conclusion and Discussion

Interesting results were obtained from the computations of electric fields from lightning using the concept of electrical image. If the positive charge is developed at the clouds negative charges gets induced at the ground , the electric field in such case gets directed in downward direction. Similarly, if negative charge gets developed at the clouds electric fields at ground gets directed in upward direction. Only the vertical component of electric field due to positive and negative charges contributes to the total electric field. Variation of electric field with distance was determined in the tri-pole structure of the cloud distribution. The graph plotted in figure 5 as a function of distance showed that the electric field reverses its sign with range: it is negative at close distance and positive faraway. This distance is called reversal distance.

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